NAG Fortran Library Routine Document

G13ACF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G13ACF calculates partial autocorrelation coefficients given a set of autocorrelation coefficients. It also calculates the predictor error variance ratios for increasing order of finite lag autoregressive predictor, and the autoregressive parameters associated with the predictor of maximum order.

2 Specification

```
SUBROUTINE G13ACF(R, NK, NL, P, V, AR, NVL, IFAIL)
INTEGER            NK, NL, NVL, IFAIL
real               R(NK), P(NL), V(NL), AR(NL)
```

3 Description

The data consist of values of autocorrelation coefficients $r_1, r_2, \ldots, r_K$, relating to lags $1, 2, \ldots, K$. These will generally (but not necessarily) be sample values such as may be obtained from a time series x_t using G13ABF.

The partial autocorrelation coefficient at lag l may be identified with the parameter $p_{l,l}$ in the autoregression

$$x_t = c_l + p_{l,1}x_{t-1} + p_{l,2}x_{t-2} + \cdots + p_{l,l}x_{t-l} + e_{l,t}$$

where $e_{l,t}$ is the predictor error.

The first subscript l of $p_{l,l}$ and $e_{l,t}$ emphasises the fact that the parameters will in general alter as further terms are introduced into the equation (i.e., as l is increased).

The parameters are determined from the autocorrelation coefficients by the Yule–Walker equations

$$r_i = p_{l,1}r_{i-1} + p_{l,2}r_{i-2} + \ldots + p_{l,l}r_{i-l}, \quad i = 1, 2, \ldots, l$$

taking $r_j = r_{|j|}$ when $j < 0$, and $r_0 = 1$.

The predictor error variance ratio $v_l = \mathrm{var}(e_{l,t})/\mathrm{var}(x_t)$ is defined by

$$v_l = 1 - p_{l,1}r_1 - p_{l,2}r_2 - \cdots - p_{l,l}r_l.$$

The above sets of equations are solved by a recursive method (the Durbin–Levinson algorithm). The recursive cycle applied for $l = 1, 2, \ldots, (L-1)$, where L is the number of partial autocorrelation coefficients required, is initialised by setting $p_{1,1} = r_1$ and $v_1 = 1 - r_1^2$.

Then

$$\begin{aligned} p_{l+1,l+1} &= (r_{l+1} - p_{l,1}r_l - p_{l,2}r_{l-1} - \cdots - p_{l,l}r_1)/v_l \\ p_{l+1,j} &= p_{l,j} - p_{l+1,l+1}p_{l,l+1-j}, \quad j = 1, 2, \ldots, l \\ v_{l+1} &= v_l(1 - p_{l+1,l+1})(1 + p_{l+1,l+1}). \end{aligned}$$

If the condition $|p_{l,l}| \geq 1$ occurs, say when $l = l_0$, it indicates that the supplied autocorrelation coefficients do not form a positive-definite sequence (see Hannan (1960)), and the recursion is not continued. The autoregressive parameters are overwritten at each recursive step, so that upon completion the only available values are p_{Lj}, for $j = 1, 2, \ldots, L$, or $p_{l_0-1,j}$ if the recursion has been prematurely halted.

4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

Durbin J (1960) The fitting of time series models *Rev. Inst. Internat. Stat.* **28** 233

Hannan E J (1960) *Time Series Analysis* Methuen

5 Parameters

1: R(NK) – ***real*** array *Input*

On entry: the autocorrelation coefficient relating to lag k, for $k = 1, 2, \ldots, K$.

2: NK – INTEGER *Input*

On entry: the number of lags, K. The lags range from 1 to K and do not include zero.

Constraint: NK > 0.

3: NL – INTEGER *Input*

On entry: the number of partial autocorrelation coefficients required, L.

Constraint: $0 <$ NL $\leq$ NK.

4: P(NL) – ***real*** array *Output*

On exit: P(l) contains the partial autocorrelation coefficient at lag l, $p_{l,l}$, for $l = 1, 2, \ldots,$ NVL.

5: V(NL) – ***real*** array *Output*

On exit: V(l) contains the predictor error variance ratio v_l, for $l = 1, 2, \ldots,$ NVL.

6: AR(NL) – ***real*** array *Output*

On exit: the autoregressive parameters of maximum order, i.e., p_{Lj} if IFAIL $= 0$, or $p_{l_0-1,j}$ if IFAIL $= 3$, for $j = 1, 2, \ldots,$ NVL.

7: NVL – INTEGER *Output*

On exit: the number of valid values in each of P, V and AR. Thus in the case of premature termination at iteration l_0 (see Section 3), NVL is returned as $l_0 - 1$.

8: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, NK ≤ 0,
or NL ≤ 0,
or NK < NL.

IFAIL = 2

On entry, the autocorrelation coefficient of lag 1 has an absolute value greater than or equal to 1.0; no recursions could be performed.

IFAIL = 3

Recursion has been prematurely terminated; the supplied autocorrelation coefficients do not form a positive-definite sequence (see Section 3). Parameter NVL returns the number of valid values computed.

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The time taken by the routine is proportional to $(\mathrm{NVL})^2$.

9 Example

In the example below the input series is the set of 10 sample autocorrelation coefficients derived from the original series of sunspot numbers by G13ABF example program. The results show 5 values of each of the three output arrays: partial autocorrelation coefficients, predictor error variance ratios and autoregressive parameters. All of these were valid.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G13ACF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NLMAX, NKMAX
      PARAMETER        (NLMAX=5,NKMAX=10)
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, NK, NL, NVL
*     .. Local Arrays ..
      real             AR(NLMAX), P(NLMAX), R(NKMAX), V(NLMAX)
*     .. External Subroutines ..
      EXTERNAL         G13ACF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'G13ACF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      READ (NIN,*) NK, NL
      WRITE (NOUT,*)
      IF (NL.GT.0 .AND. NL.LE.NLMAX .AND. NK.GT.0 .AND. NK.LE.NKMAX)
     +    THEN
         READ (NIN,*) (R(I),I=1,NK)
         IFAIL = 1
*
         CALL G13ACF(R,NK,NL,P,V,AR,NVL,IFAIL)
*
         IF (IFAIL.NE.0) THEN
```

```
            WRITE (NOUT,99999) 'G13ACF fails. IFAIL = ', IFAIL
            WRITE (NOUT,*)
         END IF
         IF (IFAIL.EQ.3) THEN
            WRITE (NOUT,99998) '   Only', NVL,
     +        'valid sets were generated'
            WRITE (NOUT,*)
         END IF
         IF (IFAIL.EQ.0 .OR. IFAIL.EQ.3) THEN
            WRITE (NOUT,*)
     +        'Lag  Partial    Predictor error  Autoregressive'
            WRITE (NOUT,*) '     autocorrn  variance ratio   parameter'
            WRITE (NOUT,*)
            WRITE (NOUT,99997) (I,P(I),V(I),AR(I),I=1,NVL)
         END IF
      END IF
      STOP
*
99999 FORMAT (1X,A,I1)
99998 FORMAT (1X,A,I2,A)
99997 FORMAT (1X,I2,F9.3,F16.3,F14.3)
      END
```

9.2 Program Data

```
G13ACF Example Program Data
 10  5
    0.8004    0.4355    0.0328   -0.2835   -0.4505
   -0.4242   -0.2419   -0.0550    0.3783    0.5857
```

9.3 Program Results

```
 G13ACF Example Program Results

 Lag  Partial    Predictor error  Autoregressive
      autocorrn  variance ratio   parameter

  1    0.800          0.359         1.108
  2   -0.571          0.242        -0.290
  3   -0.239          0.228        -0.193
  4   -0.049          0.228        -0.014
  5   -0.032          0.228        -0.032
```